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FREE, PLANE, LAMINAR JET OF AN INCOMPRESSIBLE FLUID WHOSE VISCOSITY IS EQUAL TO THE MAGNETIC DIFFUSIVITY

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FREE, PLANE, LAMINAR JET OF AN INCOMPRESSIBLE FLUID WHOSE VISCOSITY IS EQUAL TO THE MAGNETIC DIFFUSIVITY

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ABSTRACT

As a continuation of the examination of a free jet of a fluid of very high magnetic diffusivity, discussed in previous papers (Refs. 1,2) a solution is presented here for the case in which the viscosity is equal to the magnetic diffusivity. Both the case in which the magnetic field is transverse to the axis of the jet and the case in which it is longitudinal are considered.

This problem is governed by a system of classic equations comprising the equation of continuity, Navier's equations, and Maxwell's equations. The problem is examined under the conditions imposed by the following limits:

$$\left.\begin{aligned} u(y=\infty) &= 0 \\ v(y=0) &= 0 \\ \frac{\partial u}{\partial y}(y=0) &= 0 \end{aligned}\right\} \quad \text{and} \quad \left\{\begin{aligned} &\left.\begin{aligned} B_x(y=\infty) &= 0 \\ B_x(y=0) &= 0 \end{aligned}\right\} \\ &\left.\begin{aligned} B_x(y=\infty) &= 0 \\ B_y(y=0) &= 0 \end{aligned}\right\} \end{aligned}\right.$$

if the field is transverse

if the field is longitudinal

To the best of our knowledge, only the case of a longitudinal field along the axis, has been considered to the present, except for fluids with a very high magnetic diffusivity. This is indeed the only possible case for a fluid that is a perfect conductor; but, for a fluid of finite conductivity both cases are possible.

We are seeking a related solution, i.e., one of the form

$$\left\{\begin{aligned} u &= u_0(x)\, f(\eta) \\ B_x &= B_1(x)\, g(\eta) \end{aligned}\right\} \quad \text{with} \qquad (1)$$

By introducing the primitives f and g (F and G), and by the classic approximation of the boundary layer, the general system resolves itself

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into the following equations in F and G:

$$-FF'' + \frac{u_0' l}{u_0 l'}(F'^2 - FF'') = \frac{\nu}{u_0 l l'} F''' + \frac{\sigma\lambda B_1^2}{\rho u_0^2}\left[\frac{B_1' l}{B_1 l'}(G'^2 - GG'') - GG''\right], \tag{2}$$

$$G'' + \frac{l(u_0 l)'}{\lambda} FG' - \frac{u_0 l (B_1 l)'}{B_1 \lambda} F'G) = 0. \tag{3}$$

Taking into account the limiting conditions, it is easy to show that it is necessary and sufficient, in order for this system to have a solution that does not depend on η, for the coefficients $u_0' l/u_0 l'$, $\nu/u_0 l l'$, and to be constants. This condition leads by means of scales of velocity, length, and magnetic field to the following functional forms:

$$u_0 K x^m, \qquad l^2 = \alpha \frac{\nu x}{u_0} \quad \text{and} \quad B_1^2 = \beta \frac{\rho u_0^2}{\sigma\lambda}. \tag{4}$$

The system (2)-(3) then becomes

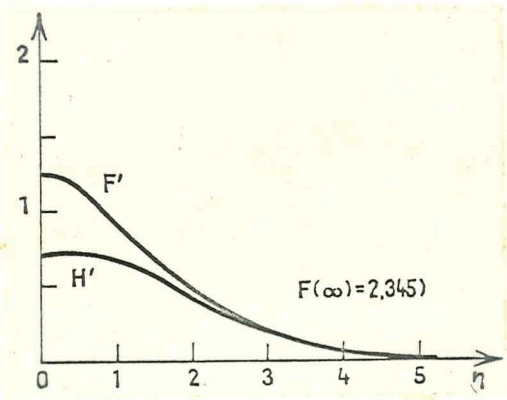
$$\frac{1}{\alpha} F''' + \frac{1+m}{2}(FF')' - \frac{1+3m}{2} F'^2 = \beta\left[\frac{1+m}{2}(GG')' - \frac{1+3m}{2} G'^2\right], \tag{5}$$

$$\frac{1}{\alpha} G'' + \frac{\nu}{\lambda}\frac{1+m}{2}[FG' - F'G] = 0. \tag{6}$$

Integrating equation (5) between zero and infinity, we obtain

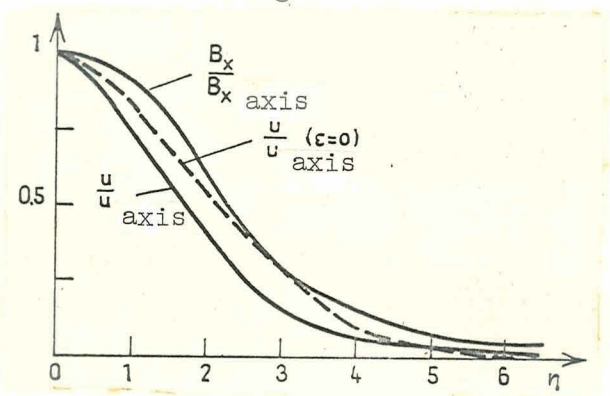
$$\frac{1+3m}{2}\int_0^\infty F'^2\, d\eta = \frac{1+3m}{2}\int_0^\infty \beta G'^2\, d\eta. \tag{7}$$

When a magnetic field is not applied, or when the fluid is a non-conductor, this equation establishes the value of m : m = — 1/3. Here, for $\beta \neq 0$, this same value (m = —1/3) also satisfies equation (7). Another possibility appears, however; if the two integrals were equal, the value of m would be indeterminate.



$\epsilon = 1$, Longitudinal field

Figure A



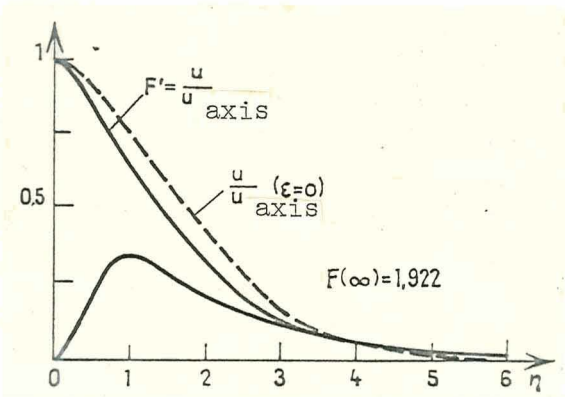
$\epsilon = 1$, Longitudinal field

Figure B

In the case of a fluid that is a perfect conductor, the solution of equation (6) is elementary: G = KF. Equation (6) requires then that

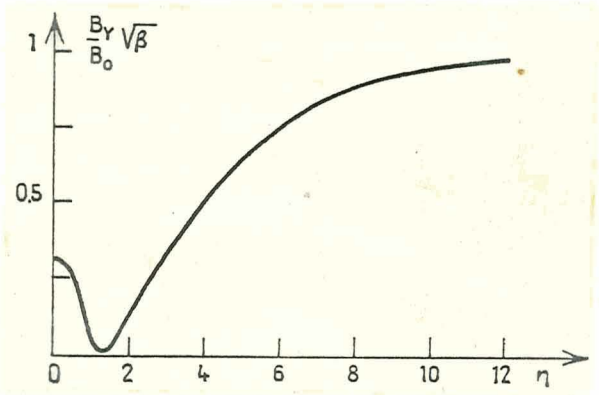
$1 - \beta K^2 \neq 0$, which is to say that the two integrals should not be equal, so that this possiblity must be rejected. To assure the continuity of the solution according to the values of ν/λ, it is tempting also to reject it in the general case. Physical considerations show, moreover, that this possibility, which demands that the flow be superalfvenic at the center and subalfvenic along the borders, is very improbable. We

adopt, therefore, the value $m = -1/3$, but we have not proved mathematically that the other possibility must be rejected.



$\epsilon = 1$, Transverse field

Figure C



$\epsilon = 1$, Transverse field

Figure D

In choosing $\alpha = 3$, and in assuming $\varepsilon = \nu/\lambda$ and $H = \beta^{1/2}G$, the system (5)-(6) may be written

$$\frac{dE'}{d\eta} + FF' = HH', \tag{8}$$

$$\frac{dH'}{d\eta} + \varepsilon FH' = \varepsilon HF'. \tag{9}$$

For any given value of ϵ, it is possible to envisage the numerical integration of the system, attempting to find an even function (the case

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of the transverse field) or an odd function (the case of the longitudinal field) for $H(\eta)$. At the same time, the value $\epsilon = 1$ leads to an interesting simplification of the calculation. In addition, it is interesting also from the physical point of view, since it corresponds to the case in which the dissipative effects are of the same order in the field of both velocity and magnetism.

Introducing the primitives of F and H, which are respectively $\varphi(\eta)$ and $\psi(\eta)$, the solution for $\epsilon = 1$ is written

$$F' = \Lambda e^{-\varphi} \operatorname{ch} \psi. \tag{10}$$

$$H' = \Lambda e^{-\varphi} \operatorname{sh} \psi. \tag{11}$$

These relations (exact) show that this flow will always be super-alfvenic ($F^1 > H^1$, or $u^2 > a_x^2$). For their integration, we have used the saddle-point method that has been applied by Meksyn (Ref. 3) for the integration of boundary layer equations. In the form of (10)-(11), they lend themselves admirably to this method.

The results are shown in the accompanying graphs.

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(2) Moreau, R., Comptes rendus, 256, 1963, p. 4849
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